```
L ::= s ; L
    | epsilon
s ::= e
    | i := e
    | e . i := e
    | { L }
    | while e do { L }
    | if e then { L } else { L }
    | skip
e ::= e * e
    | e >= e
    | e ( e )
    | e . i
    | v | o | i
    | if e then e else e
    | ( e )
v ::= n | b | f | l
n ::= (number)
b ::= true | false
o ::= [ M ]
M ::= m ; M | epsilon
m ::= i = v
f ::= fun i : T \{ L \}
l ::= (label)
i ::= (identifier)
T ::= (type)
```

`skip` and `l` exist only in the semantics, not in the user language.

Boring rules:

$$\frac{\langle L, \{\}, \{\}\rangle \to^* \langle \epsilon, -, -\rangle}{L \to^* \epsilon}$$

$$\frac{\langle L, \{\}, \{\}\rangle \to^\omega}{L \to^\omega}$$

$$\frac{\langle s_1, \sigma_1, \Sigma_1\rangle \to \langle s_2, \sigma_2, \Sigma_2\rangle}{\langle s_1; L, \sigma_1, \Sigma_1\rangle \to \langle s_2, \sigma_2, \Sigma_2\rangle}$$

$$\frac{x \in v \vee x = skip}{\langle x; L, \sigma, \Sigma\rangle \to \langle L, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_1, \sigma, \Sigma\rangle \to \langle e_2, \sigma, \Sigma\rangle}{\langle i := e_1, \sigma, \Sigma\rangle \to \langle i := e_2, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_2, \sigma, \Sigma\rangle \to \langle e_3, \sigma, \Sigma\rangle}{\langle e_1.i := e_2, \sigma, \Sigma\rangle \to \langle e_3.i := e_2, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_2, \sigma, \Sigma\rangle \to \langle e_4, \sigma, \Sigma\rangle}{\langle v.i := e_2, \sigma, \Sigma\rangle \to \langle v.i := e_4, \sigma, \Sigma\rangle}$$

$$\frac{\langle L_1, \sigma_1, \Sigma_1\rangle \to \langle L_2, \sigma_2, \Sigma_2\rangle}{\langle \{L_1\}, \sigma_1, \Sigma_1\rangle \to \langle \{L_2\}, \sigma_2, \Sigma_2\rangle}$$

$$\frac{}{\langle \{\epsilon\}, \sigma, \Sigma\rangle \to \langle skip, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_1, \sigma, \Sigma\rangle \to \langle e_2, \sigma, \Sigma\rangle}{\langle \text{if } e_1 \text{ then}\{L_1\} \text{ else}\{L_2\}, \sigma, \Sigma\rangle \to \langle \text{if } e_2 \text{ then}\{L_1\} \text{ else}\{L_2\}, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_1, \sigma, \Sigma\rangle \to \langle e_3, \sigma, \Sigma\rangle}{\langle e_1 * e_2, \sigma, \Sigma\rangle \to \langle e_3 * e_2, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_2, \sigma, \Sigma\rangle \to \langle e_4, \sigma, \Sigma\rangle}{\langle v * e_2, \sigma, \Sigma\rangle \to \langle v * e_4, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_1, \sigma, \Sigma\rangle \to \langle e_3, \sigma, \Sigma\rangle}{\langle e_1 >= e_2, \sigma, \Sigma\rangle \to \langle e_3 >= e_2, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_2, \sigma, \Sigma\rangle \to \langle e_4, \sigma, \Sigma\rangle}{\langle v >= e_2, \sigma, \Sigma\rangle \to \langle v >= e_4, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_1, \sigma, \Sigma\rangle \to \langle e_3, \sigma, \Sigma\rangle}{\langle e_1(e_2), \sigma, \Sigma\rangle \to \langle e_3(e_2), \sigma, \Sigma\rangle}$$

$$\frac{\langle e_2, \sigma, \Sigma\rangle \to \langle e_4, \sigma, \Sigma\rangle}{\langle v(e_2), \sigma, \Sigma\rangle \to \langle v(e_4), \sigma, \Sigma\rangle}$$

$$\frac{\langle e_1, \sigma, \Sigma\rangle \to \langle e_2, \sigma, \Sigma\rangle}{\langle e_1.i, \sigma, \Sigma\rangle \to \langle e_2.i, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_1, \sigma, \Sigma\rangle \to \langle e_4, \sigma, \Sigma\rangle}{\langle \text{if } e_1 \text{ then } e_2 \text{ else } e_3, \sigma, \Sigma\rangle \to \langle \text{if } e_4 \text{ then } e_2 \text{ else } e_3, \sigma, \Sigma\rangle}$$

$$\frac{\langle e_1, \sigma, \Sigma\rangle \to \langle e_2, \sigma, \Sigma\rangle}{\langle (e_1), \sigma, \Sigma\rangle \to \langle (e_2), \sigma, \Sigma\rangle}$$

$$\frac{}{\langle (v), \sigma, \Sigma\rangle \to \langle v, \sigma, \Sigma\rangle}$$

Interesting rules:

$$\frac{\sigma_2 = \sigma_1 + \{i \mapsto v\}}{\langle i := v, \sigma_1, \Sigma\rangle \to \langle skip, \sigma_2, \Sigma\rangle}$$

$$\frac{\Sigma_1(l) = [M_1] \qquad M_1(i) = v_1 \qquad M_2 = M_1 + \{i \mapsto v_2\} \qquad \Sigma_2 = \Sigma_1 + \{l \mapsto M_2\}}{\langle l.i := v_2, \sigma, \Sigma_1\rangle \to \langle skip, \sigma, \Sigma_2\rangle}$$

$$\frac{}{\langle \text{while}\, e\, \text{do}\{L\}, \sigma, \Sigma\rangle \to \langle \text{if}\, e\, \text{then}\{L; \text{while}\, e\, \text{do}\{L\}\}\, \text{else}\{skip\}, \sigma, \Sigma\rangle}$$

$$\frac{}{\langle \text{if true then}\{L_1\}\, \text{else}\{L_2\}, \sigma, \Sigma\rangle \to \langle \{L_1\}, \sigma, \Sigma\rangle}$$

$$\frac{}{\langle \text{if false then}\{L_1\}\, \text{else}\{L_2\}, \sigma, \Sigma\rangle \to \langle \{L_2\}, \sigma, \Sigma\rangle}$$

$$\frac{n_3 = n_1 * n_2}{\langle n_1 * n_2, \sigma, \Sigma\rangle \to \langle n_3, \sigma, \Sigma\rangle}$$

$$\frac{n_1 >= n_2}{\langle n_1 >= n_2, \sigma, \Sigma\rangle \to \langle \text{true}, \sigma, \Sigma\rangle}$$

$$\frac{n_1 < n_2}{\langle n_1 >= n_2, \sigma, \Sigma\rangle \to \langle \text{false}, \sigma, \Sigma\rangle}$$

$$\frac{}{\langle \text{fun}\, i : T\{e\}(v), \sigma, \Sigma\rangle \to \langle \{v/i\}e, \sigma, \Sigma\rangle}$$

$$\frac{\Sigma(l) = [M] \qquad M(i) = v}{\langle l.i, \sigma, \Sigma\rangle \to \langle v, \sigma, \Sigma\rangle}$$

$$\frac{\sigma(i) = v}{\langle i, \sigma, \Sigma\rangle \to \langle v, \sigma, \Sigma\rangle}$$

$$\frac{}{\langle \text{if true then}\, e_1\, \text{else}\, e_2, \sigma, \Sigma\rangle \to \langle e_1, \sigma, \Sigma\rangle}$$

$$\frac{}{\langle \text{if false then}\, e_1\, \text{else}\, e_2, \sigma, \Sigma\rangle \to \langle e_2, \sigma, \Sigma\rangle}$$

$$\frac{l\, \text{fresh in}\, \Sigma_1 \qquad \Sigma_2 = \Sigma_1 + \{l \mapsto [M]\}}{\langle [M], \sigma, \Sigma_1\rangle \to \langle l, \sigma, \Sigma_2\rangle}$$

Typing language and judgment (subtyping not shown). Note that this type system will reject quite a few programs that would run, some of which are potentially useful!

```
T ::= Top
    | number
    | boolean
    | T -> T
    | [ TM ]
```

```
TM ::= i : T ; TM
     | epsilon
```

$$\frac{\{\} \vdash L : T}{L : T}$$

$$\frac{}{\Gamma \vdash \epsilon : Top}$$

$$\frac{\Gamma \vdash e : T \qquad \Gamma \vdash L : Top}{\Gamma \vdash e; L : Top}$$

$$\frac{\Gamma \vdash e : T \qquad i \notin \Gamma \qquad (\Gamma + \{i \mapsto T\}) \vdash L : Top}{\Gamma \vdash (i := e; L) : Top}$$

$$\frac{\Gamma \vdash e : T_1 \qquad \Gamma(i) = T_2 \qquad T_1 <: T_2 \qquad \Gamma \vdash L : Top}{\Gamma \vdash (i := e; L) : Top}$$

$$\frac{\begin{array}{c}\Gamma \vdash e_1 : T_1 \\ \Gamma \vdash e_2 : T_2 \qquad T_1 <: [i : T_2] \qquad \Gamma \vdash L : Top\end{array}}{\Gamma \vdash (e_1.i := e_2; L) : Top}$$

$$\frac{\Gamma \vdash L_1 : Top \qquad \Gamma \vdash L_2 : Top}{\Gamma \vdash \{L_1\}; L_2 : Top}$$

$$\frac{\Gamma \vdash e : \text{boolean} \qquad \Gamma \vdash L : Top}{\Gamma \vdash \text{while}\, e\, \text{do}\{L\} : Top}$$

$$\frac{\Gamma \vdash e : \text{boolean} \qquad \Gamma \vdash L_1 : Top \qquad \Gamma \vdash L_2 : Top}{\Gamma \vdash \text{if}\, e\, \text{then}\{L_1\}\, \text{else}\{L_2\} : Top}$$

$$\frac{\Gamma \vdash e_1 : \text{number} \qquad \Gamma \vdash e_2 : \text{number}}{\Gamma \vdash e_1 * e_2 : \text{number}}$$

$$\frac{\Gamma \vdash e_1 : \text{number} \qquad \Gamma \vdash e_2 : \text{number}}{\Gamma \vdash e_1 >= e_2 : \text{boolean}}$$

$$\frac{\Gamma \vdash e_1 : T_1 \to T_2 \qquad \Gamma \vdash e_2 : T_3 \qquad T_3 <: T_1}{\Gamma \vdash e_1(e_2) : T_2}$$

$$\frac{\Gamma \vdash e : [TM] \qquad TM(i) = T}{\Gamma \vdash e.i : T}$$

$$\frac{TM = \{i : T | \Gamma \vdash M(i) : T\}}{\Gamma \vdash [M] : [TM]}$$

$$\frac{\Gamma(i) = T}{\Gamma \vdash i : T}$$

$$\frac{\begin{array}{c}\Gamma \vdash e_1 : \text{boolean} \\ \Gamma \vdash e_2 : T_1 \qquad \Gamma \vdash e_3 : T_2 \qquad T_1 <: T_2\end{array}}{\Gamma \vdash \text{if}\, e_1\, \text{then}\, e_2\, \text{else}\, e_3 : T_2}$$

$$\frac{\begin{array}{c}\Gamma \vdash e_1 : \text{boolean} \\ \Gamma \vdash e_2 : T_1 \qquad \Gamma \vdash e_3 : T_2 \qquad T_2 <: T_1\end{array}}{\Gamma \vdash \text{if}\, e_1\, \text{then}\, e_2\, \text{else}\, e_3 : T_1}$$

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash (e) : T}$$

$$\frac{}{\Gamma \vdash n : \text{number}}$$

$$\frac{}{\Gamma \vdash b : \text{boolean}}$$

$$\frac{(\Gamma + \{i \mapsto T_1\}) \vdash L : T_2}{\Gamma \vdash \text{fun}\, i : T_1\{L\} : T_1 \to T_2}$$